Mathematics in Indigenous Languages – Extending the scope of bilingual education in the NT

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I acknowledge the Larrakia as the traditional owners of the land on which we meet and pay my respects to Elders past and present.

Mathematics in Indigenous Languages (MiIL) Project

Identifying mathematical expression for teaching and learning mathematics in diverse Australian Indigenous languages – research project funded by Charles Darwin University CSFP 2021

CDU Human Research Ethics Committee clearance number H22078

Chief Investigator: Dr Cris Edmonds -Wathen

Team:

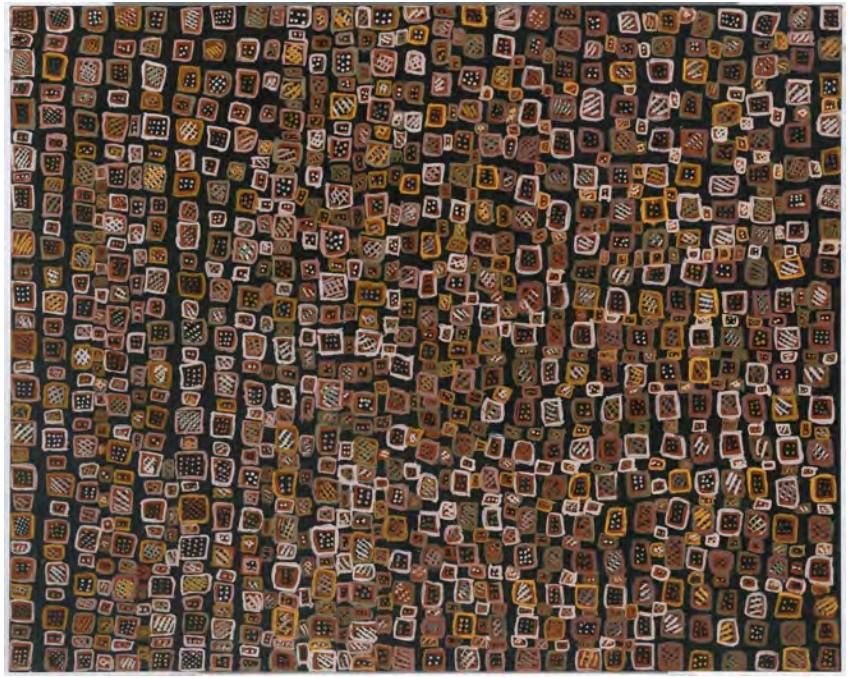
- Dr James Bednall, Charles Darwin University
- Dr Sasha Wilmoth, University of Melbourne
- Dr Sally Hughes, Charles Darwin University
- Ms Kate Charlwood, University of Melbourne/ Charles Darwin University

The project team is working with three language communities/schools to develop early primary mathematics teaching sequences in their languages

What is mathematics?

- Mathematics is about seeking patterns and relationships, representing these, symbolising these ideas, and eventually learning to abstract and generalise.

(Mason cited in Bobis, Mulligan & Lowry, 2013, p. 6)



Jean Baptiste Apuatimi, *Jikapayinga,* 2007, natural earth pigments and binder on canvas, 160.0 x 200.0 cm, National Gallery of Australia, Purchased 2007.

What is mathematics?

Looking around and thinking, “How is this always behaving in this way?” Maybe I can tell a story correctly to make it clear, “This is always behaving like this, maybe this thing and this other thing are related like family.”

And then going along looking at other things and thinking, “Oh yes, it’s the same story, lots of things are behaving like this too.”

Through one story, many things become clear.

Paranyakula kulini, “Nyangatja yaaltji-yaaltji alatjirinyi?” Tjinguṟu ngayulu tjukurpa tjukaṟurungku wangkanytjaku, utintjaku, “Nyangatja rawa alatjiripai, tjinguṟu nyangatja munu nyanga kutjupa walytjaṟara nguwanpa ngaṟanyi.”

Munu palulanguṟu kutjupa-kutjupa tjuṯa paranyakukatinyi munu kulini, “Munta uwa, tjukurpa lipula, kutjupa-kutjupa tjuṯa rawa alatjiripai kuḻu.”

Tjukurpa kutjuwanu kutjupa-kutjupa tjuṯa utiringkupai.

(Sasha Wilmoth)

Why teach mathematics in Australian languages?

Cognitive benefits:

- Children learn best in their first languages (Cummins 1979; Clarkson 2007)
- Big part of mathematics education is about learning the language for doing mathematics (O'Halloran 2015); this is harder in an additional language (Silburn et al. 2011)
- Mathematical learning can be transferred to English later (??)

Cultural identity:

- Learning in first language is a human right (UNESCO 2016)
- Language a core way people identify as part of cultural group (Lambert 1977).
- Use of language in education reinforces cultural identity (Mendes 2011; Owens 2014)
- Indigenous Australians in remote communities want education that will "help young people maintain their connection to language, land and culture" (Guenther, Disbray, & Osborne 2015 p. 199)

Language maintenance:

- Most Australian languages are endangered – others could rapidly become so
- English-only education contributes to endangerment

How to teach mathematics in Australian languages? Developing mathematics registers

- Mathematics is embedded in and mutually constitutive with language (Barton 2009)
- Mathematics register: “the meanings that belong to the language of mathematics …, and that a language must express if it is being used for mathematical purposes” (Halliday 1978 p. 195)
- Variations in mathematical connotation in different languages due to different grammatical structures and terminologies (Edmonds-Wathen, Trinick and Durand-Guerrier 2016)
- Mathematics registers in different languages are different.
- But we don’t yet know:
  - How different they are
  - The significance of the differences for learners and mathematicians

Developing mathematics registers

- Grammatical structures can be developed to express necessary mathematical relations, either a slow process or as the result of language engineering (Meaney, Trinick & Fairhall 2012).
  - Can be developed in a centralised manner for a single language, eg Maori (Meaney, Trinick, and Fairhall 2012)
  - Tends to be more ad hoc in countries with diverse languages, eg PNG (Edmonds-Wathen, Owens and Bino 2019) and Australia
  - Tensions between the oral and written registers, since many Indigenous languages have not previously been written (Mendes 2007)
- We need to be able to investigate mathematical expression/mathematics registers in different languages without privileging one language over another

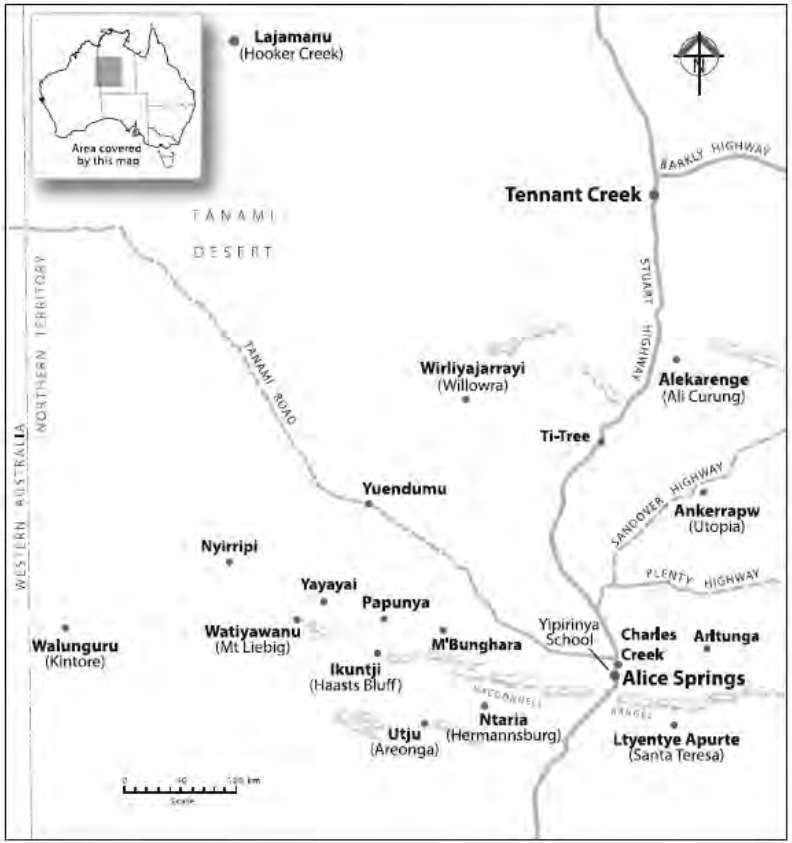
Previous work in NT on mathematics in Indigenous languages

- Timeline of commencement of bilingual education in the NT (don't have dates for cessation at schools) (Devlin et al. 2017)
- Material on languages used in bilingual schools
- Published material **with language data from languages used in NT bilingual schools** with explicit or implicit mathematical focus (eg number or space)
  - Linguistics
  - Education
  - Psychology
- School-based literature production – this is what I have been able to find to date
- Note: there is a little material available from outside the NT (eg Kimberley Kriol) or other languages (eg Iwaidja) which has not been included

Locations of bilingual schools in NT

(Devlin et al. 2017)

Map 2 Northern NT region (or 'Top End'), Australia



Year	Bilingual education commencement and key events	Published work (outside schools)	School based publications and documents
1973	Angurugu - Anindilyakwa Areyonga - Pitjantjatjara Hermannsburg - Arrernte Milingimbi - Gupapuyŋu Warruwi - Mawng		
1974	St Therese's (now Murrupurtiyanuwu) - Tiwi Shepherdson College on Galiwin'ku – Gupapuyŋu Oenpelli (Gunbalanya) - Kunwinjku Yayayi (a Papunya outstation) - Pintupi-Luritja Yirrkala - Gumatj Yuendumu - Warlpiri		
1975	Pularumpi - Tiwi Barunga and Ngukurr - oral Kriol pre-school		
1976	Barunga - Kriol Haasts Bluff - Pintupi-Luritja Numbulwar - Nunggubuyu (Wubuy) Wadeye - Murrinh Patha	Lewis, D. (1976). Observations on route finding and spatial orientation among the aboriginal peoples of the Western desert region of central Australia. *Oceania, 46*(4), 249-282. [Pintupi, Pitjantjatjara, Warlpiri]	
1977	Umbakumba - Anindilyakwa Willowra – Warlpiri some programs began to be discontinued		
1978	Maningrida - Ndjébbana	Laughren, M. N. (1978). Directional terminology in Warlpiri. In *Working Papers in Language & Linguistics no. 8* (pp. 1-16). Tasmanian College of Advanced Education.	
1979	Docker River - Pitjantjatjara		
1980		Harris, P. J. (1980). *Measurement in tribal Aboriginal communities.* Northern Territory Department of Education. [Tiwi, Iwaidja, Mawng, Rembarrnga, Wubuy (Nunggubuyu), Anindilyakwa, Murrinh-patha, Pitjantjatjara, Ngaanyatjarra, Warlpiri, Arrernte, Gurindji, Gumatj, Djambarrpuyŋu, Kriol]	
1981	M'Bunghara Homeland Centre and Watiyawanu (Mt Liebig) – Pintupi-Luritja		Laughren, M. (1981). *Number strand - Warlpiri.*
1982		Harris, J. W. (1982). Facts and fallacies of Aboriginal number systems. Work Papers of SIL [Anindilyakwa, Tiwi, Gumatj, Gurindji, Warlpiri] Stokes, J. (1982). *A description of the mathematical concepts of Groote Eylandt Aborigines*. Work Papers of SIL [Anindilyakwa]	

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1983	Walungurru (Kintore) - Pintupi/Luritja Yipirinya - Arrernte, Pitjantjatjara, Warlpiri and Western Arrernte	Rudder, J. C. (1983). *Qualitative thinking: An examination of the classificatory systems, evaluative systems and cognitive structures of the Yolngu people of Northeast Arnhem Land* [thesis, Australian National University]. Canberra. [Djambarrpuyngu]	Lefort, B., & Nguiu Ngiringawula Language Centre. (1983). *Tiwi Language maths*.
1984	Papunya - Pintupi-Luritja	Harris, P. J. (1984). *Teaching about money in tribal Aboriginal communities.* Department of Education, Professional Services Branch. [multiple per P. Harris (1980)] Harris, P. J. (1984). *Teaching about time in tribal Aboriginal communities*. Department of Education, Professional Services Branch. [multiple per P. Harris (1980)]	Warlpiri Literature Production Centre. (1984). *Pipa Nyampuju Nampapinkikirli manu Nyajangukurlu*: Bilingual Warlpiri-English Mathematics Book.
1985			
1986	Maningrida - Burarra		
1987		Harris, J. W. (1987). Australian Aboriginal and Islander mathematics. *Australian Aboriginal Studies,* 1987(2). [Anindilyakwa, Tiwi, Gumatj, Gurindji, Warlpiri]	Warlpiri Triangle Mathematics Workshops. (1987). *Karlarlakari-karlarlakari-kirli: Kujarnalu Yirri-Yuraja Manu Yirrarnu Nyurruwiyi Turnu-jarrinjarla Wirliyarrayirla manu Yurntumurla.* Warlpiri Triangle Mathematics Workshops 1985–86: The Pattern of Decimal Numeration and Its Expression in Units for Formal Measurement of Money, Length, Area, Volume and Mass. Bilingual Resource Development Unit.
1988			
1989	Ltyentye Apurte (Santa Teresa) - Eastern Arrernte		Lajamanu Maths Project Book 1 Level 0. *Translation of Rigby Maths Series.* (1989-1991). Lajamanurla manu Yurntumurla [Warlpiri]
1990		Cooke, M. (1990). *Seeing Yolngu, seeing mathematics.* [Yolngu Matha] McRoberts, R. W. (1990). *Counting at Pularumpi: a survey of a traditional mathematics and its implications for modern learning.* [Tiwi]	
1991		Ascher, M. (1991). *Ethnomathematics: a multicultural view of mathematical ideas.* Brooks/Cole. [Warlpiri] Harris, P. J. (1991). *Mathematics in a cultural context: Aboriginal perspectives on space, time and money.* Deakin University: distributed by Deakin University Press. [multiple per P. Harris (1980)]	Kerinaiua, M., Kantilla, D., & Sr Teresa Yawarntirrayamawu Ward. (c. 1991). *Tiwi mathematics syllabus.* Nguiu Ngiringawila Literature Production Centre.
1992			*Garma maths.* (1992). Yirrkala Literature Production Centre. [Gumatj] Watson-Verran, H. (1992). *We've heard that you teach maths through kinship?: A garma maths course of study in the Yirrkala and Laynhapuy Schools community.* Yirrkala Literature Production Centre. [Gumatj]

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1984	Papunya - Pintupi-Luritja	Harris, P. J. (1984). *Teaching about money in tribal Aboriginal communities.* Department of Education, Professional Services Branch. [multiple per P. Harris (1980)] Harris, P. J. (1984). *Teaching about time in tribal Aboriginal communities*. Department of Education, Professional Services Branch. [multiple per P. Harris (1980)]	Warlpiri Literature Production Centre. (1984). *Pipa Nyampuju Nampapinkikirli manu Nyajangukurlu*: Bilingual Warlpiri-English Mathematics Book.
1985			
1986	Maningrida - Burarra		
1987		Harris, J. W. (1987). Australian Aboriginal and Islander mathematics. *Australian Aboriginal Studies,* 1987(2). [Anindilyakwa, Tiwi, Gumatj, Gurindji, Warlpiri]	Warlpiri Triangle Mathematics Workshops. (1987). *Karlarlakari-karlarlakari-kirli: Kujarnalu Yirri-Yuraja Manu Yirrarnu Nyurruwiyi Turnu-jarrinjarla Wirliyarrayirla manu Yurntumurla.* Warlpiri Triangle Mathematics Workshops 1985–86: The Pattern of Decimal Numeration and Its Expression in Units for Formal Measurement of Money, Length, Area, Volume and Mass. Bilingual Resource Development Unit.
1988			*Maung maths*. (n.d).6 pages. Urabadi, R., & Jorlom, R. (n.d.). *Juka jita ninyalijap la juka jita karrkpin* [This is big and this is small]. Maung bilingual literature production centre.
1989	Ltyentye Apurte (Santa Teresa) - Eastern Arrernte		Lajamanu Maths Project Book 1 Level 0. *Translation of Rigby Maths Series.* (1989-1991). Lajamanurla manu Yurntumurla [Warlpiri]
1990		Cooke, M. (1990). *Seeing Yolngu, seeing mathematics.* [Yolngu Matha] McRoberts, R. W. (1990). *Counting at Pularumpi: a survey of a traditional mathematics and its implications for modern learning.* [Tiwi]	*Warlpiri maths checklist for lower primary.* (n.d.). 10 pages. *Warlpiri Spatial terms and sentences for Millipede to animate and make interactive games*. (n.d.). 4 pages.
1991		Ascher, M. (1991). *Ethnomathematics: a multicultural view of mathematical ideas.* Brooks/Cole. [Warlpiri] Harris, P. J. (1991). *Mathematics in a cultural context: Aboriginal perspectives on space, time and money.* Deakin University: distributed by Deakin University Press. [multiple per P. Harris (1980)]	Kerinaiua, M., Kantilla, D., & Sr Teresa Yawarntirrayamawu Ward. (c. 1991). *Tiwi mathematics syllabus.* Nguiu Ngiringawila Literature Production Centre.
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1993		Northern Territory Department of Education (1993). *Maths in Context (Maths in Aboriginal Schools), Early Childhood Units of Work: Kapi; Ngura-kutu Ankunydja* [Pitjantjatjara]	
1994			
1995			
1996			
1997			
1998	Northern Territory Government began phasing out specific purpose funding for bilingual education programs and discontinuing some programs		
1999			
2000		Angelo, D. (2000). *Space: Vocabulary & concepts relevant to Transition Oral Maths Language for teachers of Kriol speaking students.* Northern Territory Department of Education.	
2001			
2002			

Year	Bilingual education commencement and key events	Published work (outside schools)	School based publications and documents
2003			
2004			
2005	Northern Territory Government developed a strategic plan to expand the Bilingual Program		
2006		Wilkins, D. (2006). *Towards an Arrernte grammar of space.* In S. C. Levinson & D. Wilkins (Eds.), Grammars of space: Explorations in cognitive diversity (pp. 24-62). Cambridge University Press.	
2007			
2008		Butterworth, B., Reeve, R., Reynolds, F., & Lloyd, D. (2008). Numerical thought with and without words: Evidence from indigenous Australian children. *PNAS, 105*(35), 13179-13184. [Warlpiri, Anindilyakwa]	
2009	First 4 hours in English policy		*Jinta Jarrimi report*. (2009, 11-13thNovember, 2009). Jinta Jarrimi workshop, Willowra School. [Warlpiri]
2010		Jorgensen, R. (2010). Issues of social equity in access and success in mathematics learning for indigenous students. Paper presented at *Teaching mathematics? Make it count. What research shows us about effective mathematics teaching and learning,* Crown Conference Centre, Melbourne, 16-17 August. [Pitjantjatjara]	
2011			
2012		Bowern, C., & Zentz, J. (2012). Diversity in the numeral systems of Australian languages. *Anthropological Linguistics, 54*(2), 133-160. [multiple]	

Year	Bilingual education commencement and key events	Published work (outside schools)	School based publications and documents
2013		Wilkinson, M., & Bradbury, J. (2013). Number and two languages in the early years: Report on a project with paraprofessional Indigenous teachers in two NT northeast Arnhem Yolŋu schools. *Australian Review of Applied Linguistics, 36*(3), 335-354. [Djambarrpuyngu]	
2014			
2015			
2016			
2017			
2018			
2019			
2020			
2021			
2022		Edmonds-Wathen, C., & Gumurdal, J. (2022). Mawng maths: Collaborating to teach mathematics in an Australian Indigenous language. *Mathematics Education Research Journal.*	

What has been done to date?

- Lots of work at a few schools/in a few languages
- Some of this has been archived, some has been lost, little is still in use
- Little systematic or with a contemporary theoretical foundation

Mathematics in Indigenous Languages (MiIL) Project

Three typologically diverse case studies
Areyonga School:

- Areyonga/Utju (Pitjantjatjara)
- Groote Eylandt Bickerton Island Primary College Aboriginal Corporation, Groote Eylandt (Anindilyakwa)
- Murrupurtiyanuwu Catholic Primary School, Bathurst Island (Tiwi)
- All schools are:
    - Remote or very remote
    - 95-100% Indigenous students
    - 95-100% LBOTE
    - Low SES – 95-100% of students in bottom quartile

Mathematical focus - space

The project focuses on space:
shape, location, direction

- These are semantic domains with rich and precise means of expression in Australian languages (Levinson & Wilkins, 2006)
- Spatial language and basic reasoning can form a foundation for other mathematical topics (Lowrie et al., 2017; Lowrie et al., 2020; Sinclair et al., 2016)

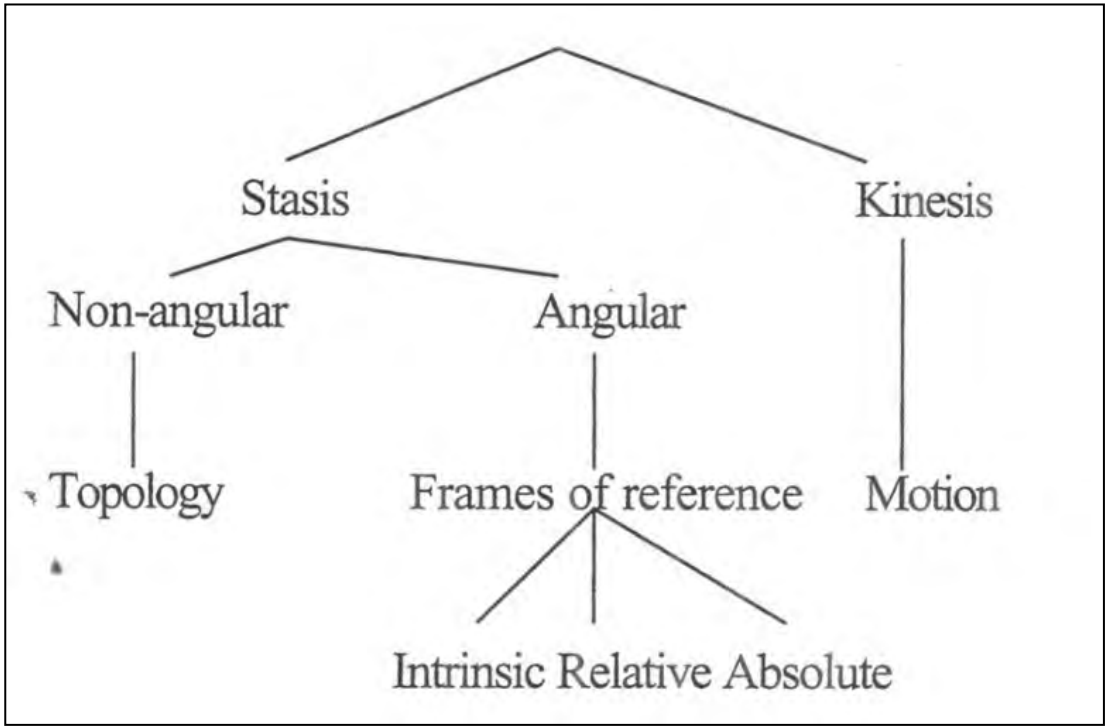
Same problem: Different solutions:

- All languages allow people to talk about space and location
- But people talk about space and location differently in different languages

Australian Curriculum: Mathematics V9.0

	Shape	Location
Foundation	sort, name and create familiar shapes; recognise and describe familiar shapes within objects in the environment, giving reasons AC9MFSP01	describe the position and location of themselves and objects in relation to other people and objects within a familiar space AC9MFSP02
Year 1	make, compare and classify familiar shapes; recognise familiar shapes and objects in the environment, identifying the similarities and differences between them AC9M1SP01	give and follow directions to move people and objects to different locations within a space AC9M1SP02
Year 2	recognise, compare and classify shapes, referencing the number of sides and using spatial terms such as “opposite”, “parallel”, “curved” and “straight” AC9M2SP01	locate positions in two-dimensional representations of a familiar space; move positions by following directions and pathways AC9M2SP02
Year 3	make, compare and classify objects, identifying key features and explaining why these features make them suited to their uses AC9M3SP01	interpret and create two-dimensional representations of familiar environments, locating key landmarks and objects relative to each other AC9M3SP02

Spatial Language



(Levinson & Wilkins, 2006)

Focus on diversity

Development of a theoretical and methodological approach that positions diverse languages on equal terms and counters deficit narratives

- Languages
- Environments
- School systems

Eg Language of comparison

- "Many Aboriginal and Torres Strait Islander languages don't have words of comparison (for example 'big mobs', 'big big mobs')." (ACARA 2014)
- Three main types of comparative structures in the world's languages: standard, implicit (or conjoined), and exceed (Beck et al., 2009)

Examples of comparative structures

- Standard comparative - English, Wambaya

  Bulyingi nyamirniji ngarra

  little.I(NOM) you.NOM I.OBL 'You're littler than me.' (Nordlinger 1998 p. 176)

  A literal translation might be "You are little to me".

- Implicit or conjoined comparative, Kunbarlang

  Watakiyi nyampu-ju yukanti, nyampu-ju wirijarlu

  bush.orange this-TOPIC small this-TOPIC big
  'This bush orange is bigger than that one.' (Bowler 2016 p.13)

  Literally translates "This bush orange is small, this one is big", but can be used for a small difference in size

  Implicit or conjoined comparative juxtaposes a positive statement about the statement with a negation or intensification (Beck et al. 2009).

Although implicit comparatives use a negation, they can still be used to make "crisp judgements" – the bush oranges were almost the same size (Bowler 2016).

- Exceed comparative, Djambarrpuyngu

  A child uses a verb meaning "to overtake, pass" to express "bigger than" with numbers. (Wilkinson and Bradbury 2013)

Typology

- A **typological** approach is useful to investigate mathematical expression in different languages without privileging one language over another
  - Typology is the area of linguistics that deals with finding, describing, and classifying languages according to their structural similarities and differences (Dixon 2010).
  - "concerned with developing a body of analytically compatible concepts … valid across all the world's languages" (Evans & Dench 2006, p. 4).
  - compare and describe languages in a framework-neutral manner (Nichols 2007)
- **Syntactic typology:** the different ways that languages structure phrases and sentences
- **Semantic typology:** the different ways that languages structure semantic domains such as time and space.

Functional perspective

Functional linguistics:

- language exists and develops to do things (Butler, 2003)
- languages can be analysed and described by how they do things

A **functional typological perspective** enables the mathematical concepts, practices and affordances of diverse languages to be investigated within a broader mathematical frame.

- Mathematics as a semantic and functional field (Edmonds-Wathen 2019)
- What functions are performed by language features that are used in existing school/academic/formal mathematics?
  - How are those functions performed in the target language?
  - Can the way that those functions are performed by drawn on or extended for the purposes of school mathematics?

Prepositions in mathematics in English

- "Some of the more challenging words in English for all EAL/D students are the small words (for example the articles 'the', 'an', 'a', and the prepositions 'on', 'under', 'over')" (ACARA 2014)
- [In English] "coming to learn mathematics is heavily associated with the use of prepositions" (Jorgensen, 2010, p. 29)
- Many have a core spatial meaning directly relevant to mathematics, metaphorically extended into other domains
  - E.g. talking about numbers occurring before or after each other in a sequence

What about languages that don't use many prepositions:

- E.g. "In Pitjantjatjara, there are less than 10 prepositions, whereas English has more than 60" (Jorgensen, 2010, p. 29)
- **Functions** of prepositions overlap with those of case markings

➢ Consider languages with case systems that already have mathematics registers
  - E.g. Finnish

Locational case (Pitjantjatjara)

The locational relation is expressed through the case suffix *-ngka* on the noun wali ‘house’

Wati tjuta walingka nyinanyi

wati	*tjuta*	*wali-ngka*	*nyina-nyi*
man	many	house-LOC	sit-PRES

‘The men are in/near/at the house’ (Goddard, 1985, p. 78)

- “an expression like *wali-ngka* ‘house-LOC’ could be used where in English we would have to choose between ‘in the house’, ‘at the house’, ‘on the house’ and so on” (Goddard, 1985, p. 78)

Similarities between Finnish and Pitjantjatjara locative cases

Selected Pitjantjatjara locative cases (Goddard 1985)

case	suffix	English preposition	example	translation of the example
nominative	–	–	*talo*	house
adessive	*-lla*	at, on	*talolla*	at (a) house
allative	*-lle*	to	*talolle*	to (a) house
elative	*-sta*	from (inside)	*talosta*	from (a) house

Selected Finnish locative cases (Korpela 2014)

case	suffix	English preposition	example	translation of the example
nominative	–	–	*wali*	house
locative	*-ngka*	in, at, on	*walingka*	at (a) house
allative	*-kutu*	to	*walikuta*	to (a) house
ablative	*-nguru*	from	*walinguru*	from (a) house

Look at languages that are typologically similar rather than compare typologically different languages

Diverse environments

- Topographic Correspondence Hypothesis
  - Aspects of a language's spatial reference system often correlate with salient topographic features of the language locus (Palmer 2015)
- Sociotopographic Model
  - The role of the environment in shaping spatial language is mediated by the nature of each individual's interaction with their environment, and other sociocultural factors (Palmer et al. 2017)
- Designing learning for ecocultural diversity (Owens Edmonds-Wathen and Bino 2015)
  - Improving the teaching of mathematics in elementary schools by using local languages and cultural practices (Papua New Guinea)

Designing learning for ecocultural diversity

Thinking mathematically

- Identifying shapes and number patterns in cultural activities and artefacts helps teachers and students understand the nature of mathematics.

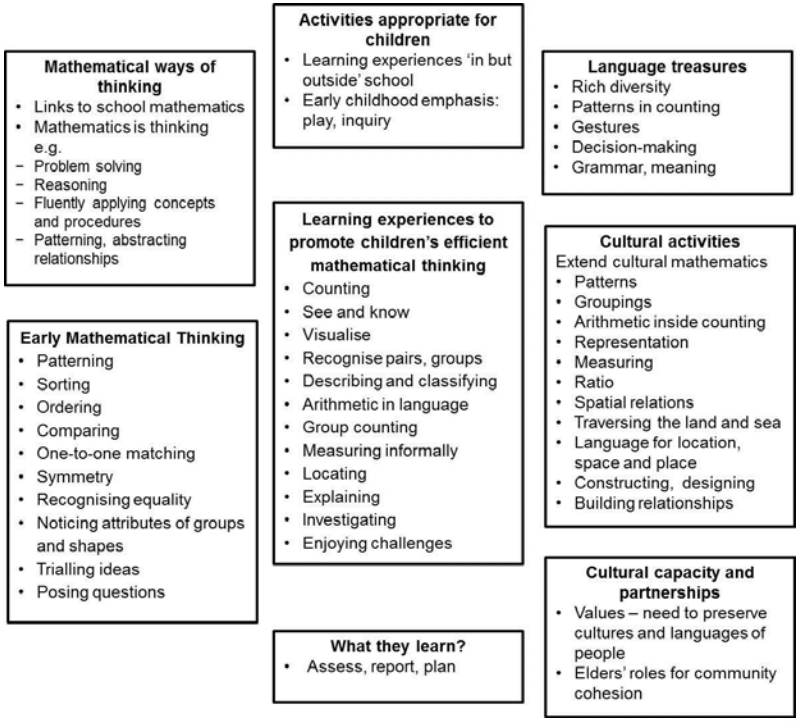
Using Tok Ples

- Teachers unpack the mathematical treasures in their own languages. They explore different words which can be used in teaching mathematics. Here teachers investigate the way number words in their own languages are created

Identifying the mathematics in cultural activities

- Building a stilt house - Teachers learn to recognise the mathematics embedded in cultural practices including weaving, construction of buildings and design of the environment. Cultural activities are used as a focus to explore mathematical concepts and develop lessons.

(Owens, Edmonds-Wathen and Bino 2015)



School:

Areyonga School

- 34 students
- Government
- Bilingual since 1973:
- recently celebrated 50 years of bilingual education

Linguist:

Sasha Wilmoth

Language:

Pitjantjatjara

- 3500 speakers
- Pama-Nyungan – Western Desert dialect
- Rich case marking system
- Free word order
- Frequent and sometimes compulsory nominalization

Environment:

Utju Community

- Central Australia
- Desert / West MacDonnell Ranges
- Strong cardinal system

Organisation:

Groote Eylandt Bickerton Island Primary College Aboriginal Corporation

- 4 Government schools & planning new boarding school
- Local Decision Making agreement - Developing community-controlled, bi-lingual education system for all schools in the Groote Archipelago
- Angurugu School was previously bilingual from 1973 until sometime in late 1990s

Language:

Anindilyakwa

- 1500 speakers
- Polysynthentic and polymorphic
- Gunwinyguan
- Rich case marking system
- Intricate systems of nominal classification

Linguist:

James Bednall

Environment:

Groote Eylandt

- East Arnhem Land - Gulf of Carpentaria
- Island
- Wind system
- Frequent use of relative left/right

School:

Murrupurtiyanuwu Catholic Primary School

- 166 Students
- Catholic Education NT
- Different funding model from gov't schools– more local staff can be employed
- Bilingual previously bilingual from 1974 until early 2000s; strong desire from Tiwi people to resume bilingual

Language:

Tiwi

- 2100 speakers
- Rapid intergenerational change
- Was previously extremely polysynthetic but is now more isolating/ analytic
- Traditional Tiwi> modern Tiwi > new Tiwi

Environment:

Wurrumiyanga, Bathurst Island

- Top End
- Island
- Little information about topographic systems – some use of left/right and sun-oriented cardinals

Linguist:

Kate Charlwood

What we are doing

- Developing learning sequences and assessments in L1
  - A Shape sequence and a Location sequence for Years F-2
- Integrating sequences into school programme
- Pre- and post-assessment of students in two languages
- Reporting on student achievement in first language

Planning for teaching

Shape: The shape sub-strand is a good place to start developing mathematical reasoning. The focus through the strand (F-3) is on classification. Shapes and objects are described, compared and classified, and students learn to give reasons for their classification.

Key mathematical concepts:

- sameness, similarity and difference
- Being a type of something
- Descriptive attributes

Key instructional language:

- Sort, put things in groups
- Identify
- Describe

Key descriptive language:

- This belongs with that because …

Cultural links may be found with:

- Natural environment – links to Location
- Built environment
- Artefacts (tools)
- Visual art – links to Location
- Arrangement of people (e.g. circling, making a straight line) – links to Location

Collaboration

- Working with elders and language specialists
  - Identification and development of mathematics lexicon
  - Consultation on linking mathematics teaching with local cultural practices
- Indigenous teaching staff – teachers and assistant teachers
  - Workshopping lesson plans and practicing teaching
  - Assessing students in L1
  - Trialling lessons in class

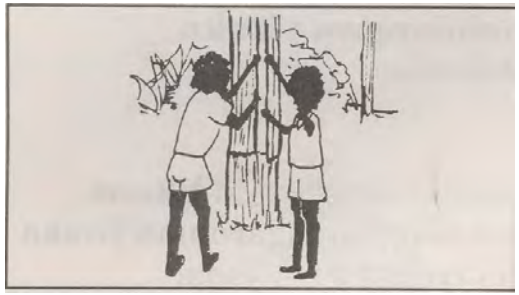
Identifying lexicon through metaphor

	SHAPE						side
Pitjantjatjara	*puntu*	*manguṟi-manguṟi*	*kanti*	*waḻu*	*waḻu wara*	*mulya*	*kampa*
	body	a head-ring used for carrying	quartz chip, used as a blade	flat rock used as a lower grindstone	long grindstone	nose; end of a rockhill ridge	side
							
Anindilyakwa	*amurndakajika*	*milyelya*	*munguwa*	*bada*	*amureba*	*amukwurra*	*emkadabuda*
	thing	armband	top shell	box	bark sheet	face	edge of a flat place
							

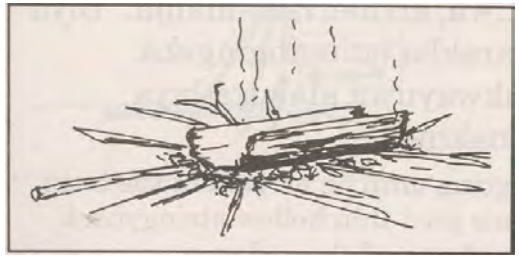
Shape through Art
amureba ‘rectangle’

(https://www.aboriginal-bark-paintings.com/aboriginal-art-gallery/groote-eylandt/)

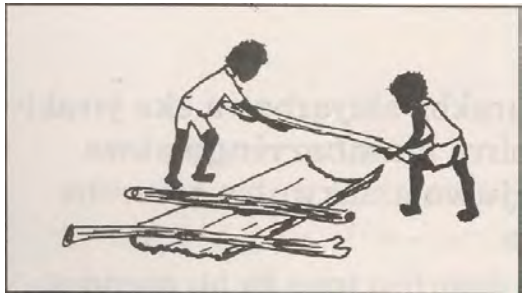
Shape through Art
amureba - bark collecting

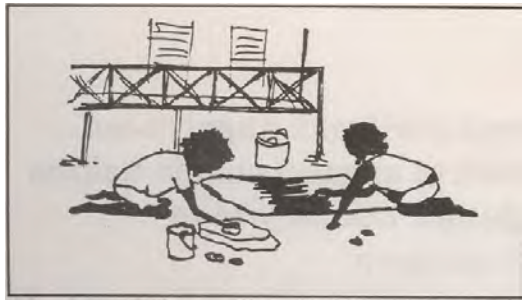


Wunambilyuma wunenungkwarba-kiya neni-ridanga akina amureba akwa neni-ngarngkuwayijuwa.
'**Two men cut the bark and pulled it off [the tree]**'



Kembirra neni-rembemkidaka karrawara angwura-manja akwa neni-lalika yakwujina na-dada.
'**They put the bark sheet on the fire and left it to get hot**'





Kembirra nenu-manga akina angwuralangwa akwa nen-akumarna ajiringkamanja neni-rerrrnaj uwa akwa nen-akumarna engkubarrngwarrngwa eka-murriya karrawara kajungwa eningabi-yada aremberrbirri-yada. Kembirra nenu-wilyaka angalyu-wa akwa nen-ambarringg akwa neni-rungkwulanga akwa neni-lyungkwena arakba akina amureba.
'**They took [the bark sheet] off the fire and put it on the ground to dry. They put heavy pieces of wood on top to make it good and flat. And they took it home and they sat down and rubbed the coloured clay on the stones and painted the bark**'

(Groote Eylandt Linguistics 1993: 180-181)

Shape through Art
amureba - bark painting

(Cross hatching from A. Lalara's 'Groote Eylandt Mission')

(Triangle motifs from A. Lalara & Alice Durrila's 'Macassan prau')

Linking planning to local topic cycle - Anindilyakwa

Topic	Lesson ideas		Mathematics focus	Possible links to Country and Culture Topic Cycle:	ILC Culture:
Constructing tools and other artefacts	• Cutting and preparing bark for paintings (amureba) • Making spears • Making milyelya – armbands • Using ochres for painting	Amureba Bark: • Making a video of people doing this to show the kids • Having a book telling how to do this (like the story in the green book pp. but with extra language about the shape of the bark) • Taking the kids on excursion to do this	Rectangles, curves, Making it flat Making it straight Making it round Circles Colours – used for sorting	• Bush plants (useful plants) • History	Understand traditional technologies and how they were and are used: • Observe/explore examples of traditional tools and artefacts. Students identify how people live on Country now and in the past: • Retell stories of the past after listening to stories and viewing photos of the old days. Students know their own and other people's relationships to Country: • Experience visits on Country as enjoyable and interesting ways to learn.
Paintings	• Learning the stories of certain paintings • Talking about the shapes in the paintings • Making their own paintings with shapes in them?? (e.g making a picture of a boat with shapes but it might not be traditional style)	– for example, pictures of Macassan praus – specific paintings if artists or artist families agree Could make a video of the artist or artist family talking about the story of the painting Is it ok to talk about the story and also talk about the shapes?	Rectangles, triangles, squares, circles Symmetry Parallel lines	• History	Understand traditional technologies and how they were and are used: • Observe/explore examples of traditional tools and artefacts. Students identify how people live on Country now and in the past: • Retell stories of the past after listening to stories and viewing photos of the old days.

Professional Development

Teaching staff participate in professional development to make sure everyone understands the area of the mathematics curriculum that we are focussing on.

Assessment in both languages

- Lesson kuwaripangka nganaṉa assessment palyaṉingi, Pitjantjatjarangka munu Englishangka.
- Ngayulu tjananya tjapiningi, “Shape nyaa nyangatja?” ka tjana wangkapai manguṟi-manguṟi, or kanti, or walu or walu waṟa.
- Ka ngula nganaṉa piṟuku assessment palyalku, tjitji tjuṯa puḻkaṟa nintiringkunyangka.

- Before we started teaching, we tested the kids in Pitjantjatjara and English.
- We ask them what the shapes are called, and they tell us the names, and how many sides and corners the shapes have.
- Later we’ll test them again to see what they’ve learned.

(Lucinda Nipper)

Planning lessons: location and direction

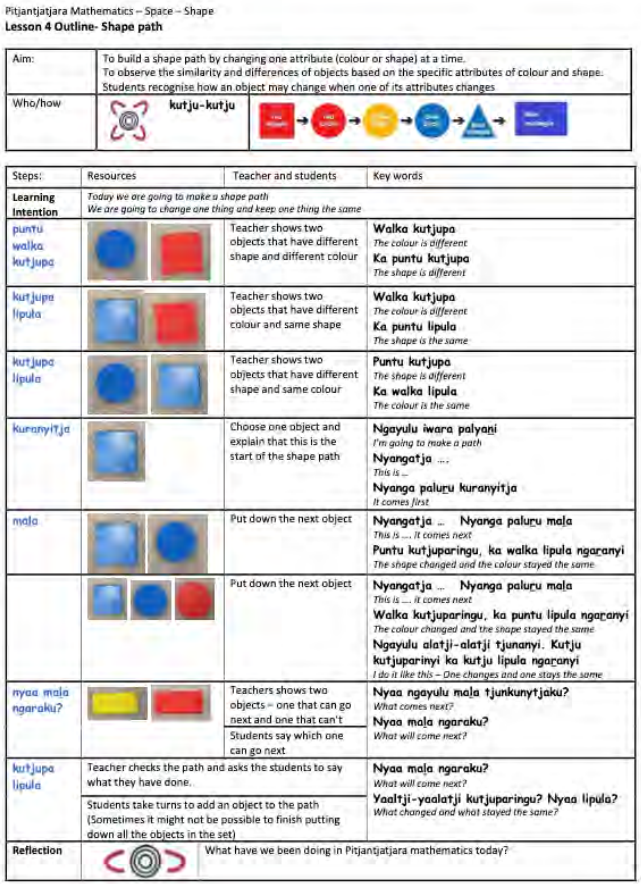
- Kuwari nganaṉa practise-arinyi, ngula tjitji tjuṯa nintilkitja, game inkanytjaku, locationku munu directionku nintiringkunytjaku. Nganaṉa ”Simon says” inkanyi, munu mat-pangka mai puṯitja uraṉi. Tjitji kulunypa tjuṯa kuwaripa nintiringanyi directionpaku panya aḻinytjara, uḻpaṟira, palumpa tjanampa kuwaripatu nintiringanyi.

- Now we’re practising these lessons to teach the kids later about direction and location.
- We’re playing games like “Simon says” and collecting bush foods on the mat.
- The little kids need to practise words like north, south, east and west.

(Lucinda Nipper)

Doing things with shapes in Pitjantjatjara

- Describing shapes:
  - *Ini wangkanyi* = Saying the name
  - *Kampa yaaltjiṯu? Mulya yaaltjiṯu?*
    How many sides? How many corners?
- Sorting into groups:
  - *Puntu walytjangka paraṟa tjunanyi*
    puntu = body/shape
    walytjangka = with own/with family
    paraṟa = dividing up
    tjunanyi = putting
- Attribute train (resolve.edu.au):
  - *Iwara tjunanyi*
    'putting down a track'
  - *Nyaa kutjupa? Nyaa lipula?*
    What's changed? What's stayed the same?



Pitjantjatjara Mathematics – Space – Shape

Lesson 4 Outline- Shape path

Aim:	To build a shape path by changing one attribute (colour or shape) at a time. To observe the similarity and differences of objects based on the specific attributes of colour and shape. Students recognise how an object may change when one of its attributes changes
Who/how	**kutju-kutju**

Steps:	Resources	Teacher and students	Key words
Learning Intention	*Today we are going to make a shape path* *We are going to change one thing and keep one thing the same*		
puntu walka kutjupa		Teacher shows two objects that have different shape and different colour	**Walka kutjupa** *The colour is different* **Ka puntu kutjupa** *The shape is different*
kutjupa lipula		Teacher shows two objects that have different colour and same shape	**Walka kutjupa** *The colour is different* **Ka puntu lipula** *The shape is the same*
kutjupa lipula		Teacher shows two objects that have different shape and same colour	**Puntu kutjupa** *The shape is different* **Ka walka lipula** *The colour is the same*
kuranyitja		Choose one object and explain that this is the start of the shape path	**Ngayulu iwara palyaṉi** *I'm going to make a path* **Nyangatja** …. *This is …* **Nyanga paluṟu kuranyitja** *It comes first*
mala		Put down the next object	**Nyangatja** … **Nyanga paluṟu mala** *This is …. It comes next* **Puntu kutjuparingu, ka walka lipula ngaṟanyi** *The shape changed and the colour stayed the same*
		Put down the next object	**Nyangatja** … **Nyanga paluṟu mala** *This is …. It comes next* **Walka kutjuparingu, ka puntu lipula ngaṟanyi** *The colour changed and the shape stayed the same* **Ngayulu alatji-alatji tjunanyi. Kutju kutjuparinyi ka kutju lipula ngaṟanyi** *I do it like this – One changes and one stays the same*
nyaa mala ngaraku?		Teachers shows two objects – one that can go next and one that can't Students say which one can go next	**Nyaa ngayulu mala tjunkunytjaku?** *What comes next?* **Nyaa mala ngaraku?** *What will come next?*
kutjupa lipula	Teacher checks the path and asks the students to say what they have done. Students take turns to add an object to the path (Sometimes it might not be possible to finish putting down all the objects in the set)		**Nyaa mala ngaraku?** *What will come next?* **Yaaltji-yaalatji kutjuparingu? Nyaa lipula?** *What changed and what stayed the same?*
Reflection		What have we been doing in Pitjantjatjara mathematics today?	

Teaching the kids in language

Shape walk

Nganaṉa tjitji tjuṯa katingu walk. Tjana shape tjuṯa nyakula picture mantjiningi. Munuya kuulakutu malaku pitjala, nyakula wangkangi puntu tjutatjara

We took the kids for a shape walk and they took photos. Back at school we talked about the shapes in the pictures

(Christine Bennett and Lucinda Nipper)

Hypernyms: superordinate terms

Instructional language depends on whether a higher order attribute term has been identified or not.

There is a word for “shape”:

Pitjantjatjara

- ***Puntu*** *palunya pu̲runypangka tjura.*
  - Put the ones with the same **shape** together.

Anindilyakwa

- *Wurrakumurna adidirribura* ***amurndakajika***
  - You (all) are to put the same **shapes** together

There is no word for “size”:

- ***Pu̲lka*** *tju̲ta tjura munu* ***ku̲lunypa*** *tju̲ta tjura.*
  - Put the **big** ones (together) and put the **small** ones (together).

- *Wurrakumurna* ***arumuruma*** *amurndakajika yelakwa*
  - You (all) are to put the big shapes here.
- *Wurrakumurna* ***eyukwayuwa*** *amurndakajika yelakwa*
  - You (all) are to put the small shapes here.

A thorny problem: what is the relationship between concepts and names?

Comparison

- We find at least three comparison strategies in Pitjantjatjara:
  - *A puḻka, B kuḻunypa*
    A is big, B is small
  - *A puḻka B-ku*
    A is big B-DAT
  - *A puḻka B-ngka munkara*
    A is big beyond B
  - *Nguwanpa* can also follow *puḻka* to mean ‘more’

Going forward

- Much more to be done on each of these languages
- More to be done to embed the program in schools
- Possible scaling up:
  - More schools (eg South Australia APY schools)
  - More languages
  - More mathematics strands
  - More year levels
- Bringing things together – what are we learning about languages in general as well as the specific languages involved?

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